HEAT TRANSFER ASSOCIATED WITH THE MOTION OF A GAS THROUGH A FIXED LOOSE PACKING WITH VARIABLE GAS TEMPERATURE AT THE INLET

V. P. Kharitonov

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 2, pp. 205–211, 1967

UDC 536.248

A solution is offered for the problem of the nonstationary temperature field in a loose packing through which passes a flow of gas with variable inlet temperature. An approximate method of calculating the gas and packing temperatures is proposed.

The process of nonstationary heat transfer associated with the motion of a gas or liquid through a loose packing can be described by a system of two linear partial differential equations [1, 2]

$$\frac{\partial T_g}{\partial t}+v\frac{\partial T_g}{\partial x}=-k_2(T_g-T_s),$$
$$\frac{\partial T_s}{\partial t}=k_1(T_g-T_s). \qquad (1)$$

In constructing these equations the following assumptions were made:

1. At any instant the temperature of a particle may be assumed constant over its entire volume, convective heat transfer between the flow and the packing is controlling; heat exchange by thermal conduction in the axial direction and by radiation is small and may be neglected; and the flow pressure remains constant during motion through the packing.
2. The flow velocity is constant; the variation of the density and thermophysical characteristics of the flow and the packing with variation in temperature may be neglected.

The problem of finding the temperatures of the gas and the packing as functions of two independent variables (time and coordinate) has been solved by Anzelius [1] and Schumann [2, 3] for the case of heating or cooling of a bed of loose packing by a constant-temperature gas flow with the following boundary conditions:

$$T_s^a(0, x)=0;\quad T_g^a(t, 0)=T_0. \qquad (2)$$

The results of Schumann's calculations were extended by Furnas [4].

In the more general case considered in this paper the temperature of the gas at the inlet to the bed is a given function $f^a(t)$ of time, and the problem of finding the temperature field reduces to solving system (1) with the boundary conditions

$$T_g^a(t, 0)=f^a(t);$$
$$T_g^a(0, x)=T_s^a(0, x)=T_0;\ f^a(0)=T_0. \qquad (3)$$

Problem (1)–(3) can be reduced to the solution of two equations of hyperbolic type. The first of these is written in the form

$$\frac{\partial^2 V}{\partial y\partial z}-V=0 \qquad (4)$$

with initial data on the characteristics

$$V(0, y)=0,$$
$$V(z, 0)=\exp z\cdot f\left(\frac{z}{k_1}\right)-\int_0^z \exp\xi\cdot f\left(\frac{\xi}{k_1}\right)d\xi. \qquad (5)$$

The second equation has the analogous form

$$\frac{\partial^2 U}{\partial y\partial z}-U=0 \qquad (6)$$

with initial data on the characteristics

$$U(0, y)=0,$$
$$U(z, 0)=\exp z\cdot f\left(\frac{z}{k_1}\right)+\int_0^z \exp\xi\, f\left(\frac{\xi}{k_1}\right)d\xi. \qquad (7)$$

The functions U(y, z) and V(y, z) are related with the relative temperatures T_g and T_s as follows:

$$T_g=0.5(U+V)\exp(-y-z),$$
$$T_s=0.5(U-V)\exp(-y-z). \qquad (8)$$

The relative temperatures T_g and T_s are expressed in terms of the absolute temperatures of the gas and the packing T_g^a and T_s^a in accordance with the formulas

$$T_g=(T_g^a-T_0)/T_0,\quad T_s=(T_s^a-T_0)/T_0. \qquad (9)$$

Solving Eqs. (4), (6) together with conditions (5), (7) by the method of adjoint differential operators and going over in accordance with formulas (8) to the unknown functions, we obtain in explicit form the final solution of problem (1)–(3), which is a generalization of the Schumann problem ($f(t)$ = const):

$$T_s=\exp(-y-z)\int_0^z \exp\xi\cdot f\left(\frac{\xi}{k_1}\right)J_0\left(2i\sqrt{(z-\xi)y}\right)d\xi,$$
$$T_g=\exp(-y)f\left(\frac{z}{k_1}\right)-\exp(-y-z)\int_0^z \exp\xi\cdot f\left(\frac{\xi}{k_1}\right)\times$$
$$\times\frac{\partial J_0\left(2i\sqrt{(z-\xi)y}\right)}{\partial\xi}d\xi. \qquad (10)$$

Equations (10) were used to calculate the nonstationary temperature field for a step boundary function equal to T_{g0}^a = const on the interval $0 < t \le t_i$ and to T_0 = const everywhere outside it. In this case as in the relative temperatures of the gas and the packing it is convenient to take the following ratios:

when $T_{g0}^a < T_0$ (cooling) $T_g=\frac{T_0-T_g^a}{T_0-T_{g0}^a},\quad T_s=\frac{T_0-T_s^a}{T_0-T_{g0}^a};$

when $T_{g0}^{a} > T_0$ (heating) $T_g = \frac{T_g^a - T_0}{T_{g0}^a - T_0}$, $T_s = \frac{T_s^a - T_0}{T_{g0}^a - T_0}$.

Then boundary conditions (3) for system (1) are written as

$$T_g(0, x) = T_s(0, x) = 0,\ T_g(0, 0) = 0;$$
$$T_g(t, 0) = 1 \ \text{at} \ 0 < t \leqslant t_i;$$
$$T_g(t, 0) = 0 \ \text{at} \ t > t_i. \tag{11}$$

Introducing the notation $\varphi(x, y) = \exp(-x - y) \cdot J_0[2i(xy)^{1/2}]$, we can transform the solution of problem (1), (11) as follows:

at $z \leq z_i$

$$T_s(y, z) = \int_0^z \varphi(x, y)\, dx, \tag{12}$$

$$T_g(y, z) = T_s(y, z) + \varphi(y, z); \tag{13}$$

at $z > z_i$

$$T_s(y, z) = \int_{z-z_i}^{z} \varphi(x, y)\, dx, \tag{14}$$

$$T_g(y, z) = T_s(y, z) + \varphi(y, z) - \varphi(y, z - z_i). \tag{15}$$

Equations (14), (15) were used to calculate the temperature field for a wide range of values of y and z at $z > z_i$. The calculations were made on an electronic computer; the program consisted in solving integral (14) by Simpson's method with forced choice of integration step depending on the required accuracy (four correct places after the decimal point). The zero-order Bessel function of the first kind was replaced with its asymptotic approximation in accordance with a formula giving seven correct places after the decimal point at $xy > 100$ [5]:

$$J_0(2i\sqrt{xy}) = \frac{\exp(2\sqrt{xy})}{\sqrt{4\pi\sqrt{xy}}}\left(1 + \frac{0.125}{2\sqrt{xy}} + \frac{0.07}{4xy}\right). \tag{16}$$

The maximum error due to replacing the Bessel function with the approximate formula does not exceed 0.00005 over the entire range of computed values.

The results of the computations in dimensionless form were tabulated for the sections $T_g(z, y_0)$ and $T_s(z, y_0)$ at y_0 from 30 to 2000 for values in the range $z_i = (0.05-0.40) \cdot y_0$, and also for the sections $T_g(z_0, y)$ and $T_s(z_0, y)$ at z_0 from 100 to 2000 for values in the range $z_i = 50-400$. The function $T_g(x, y_0)$—the temperature of the gas at the outlet from a bed of length y_0 as a function of time—which is often used in applications, is presented in graphic form in the figure. The outlet temperature curves were constructed for values of the parameter z_i/y from 0.05 to 0.40.

It follows from (13) that for heating or cooling of the packing with a flow of constant-temperature gas the difference between the temperatures of the gas and the packing is calculated in the explicit form

$$\Delta T = T_g - T_s = \varphi(y, z). \tag{17}$$

For the case of a pulsed stepwise variation of gas temperature at the inlet the temperature difference ΔT is found from Eq. (15) as follows:

$$\Delta T = \varphi(y, z) - \varphi(y, z - z_i). \tag{18}$$

Using the asymptotic approximation of the Bessel function (Eq. (16)), for not too small values of the length of the bed ($y_0 > 10$) we obtain an approximate formula for the maximum value of the function

$$\varphi_{max}(y_0, z) \cong 1/\sqrt{4\pi y_0}. \tag{19}$$

Below we give maximum values of the temperature difference between the gas and the packing for heating or cooling of the packing with a constant-temperature flow:

y_0	100	400	800	1200	1600	2000
ΔT_{max}	0.028	0.014	0.010	0.008	0.007	0.006

In the case of a stepwise variation of the gas temperature at the inlet (intermittent heating or cooling) the temperature difference varies in time according to a more complex law, but since the function $\varphi(y, z)$ is positive, the absolute value of the temperature difference given by Eq. (18) does not exceed the maximum value of the analogous difference in the case of simple heating (Eq. (17)).

For sections of the packing corresponding to a reduced length $y_0 > 100$ (the inlet region is not considered), the modulus of the temperature difference for cases of continuous and intermittent heating or cooling does not exceed 0.03, i.e., 3% of the maximum gas temperature drop at the inlet. For this reason in approximate methods of calculating the temperature field used in the design of processes and equipment it is not possible to distinguish between the temperature of the gas and that of the packing, and to introduce the similar temperature T.

It can be shown that at values of $y \leq z$ the integral (12) can be approximated by

$$I_1 \leqslant \Phi(\sqrt{2y}) - \Phi(\sqrt{2y} - \sqrt{2z}), \tag{20}$$

and the integral (14) by

$$I_2 \leqslant \Phi\left(\sqrt{2y} - \sqrt{2(z - z_i)}\right) + \Phi(\sqrt{2z} - \sqrt{2y}). \tag{21}$$

Estimates (20) and (21) are very close and the right side of the inequalities can be used as formulas for determining the temperature T.

Moreover, direct calculations have established that the temperature T thus obtained is the arithmetic mean of the temperatures of the gas and the packing, this result being valid not only at $y \leq z$ but at any values of y, z, and z_i. The deviations from this law do not exceed the error of the calculations, namely, 0.0002.

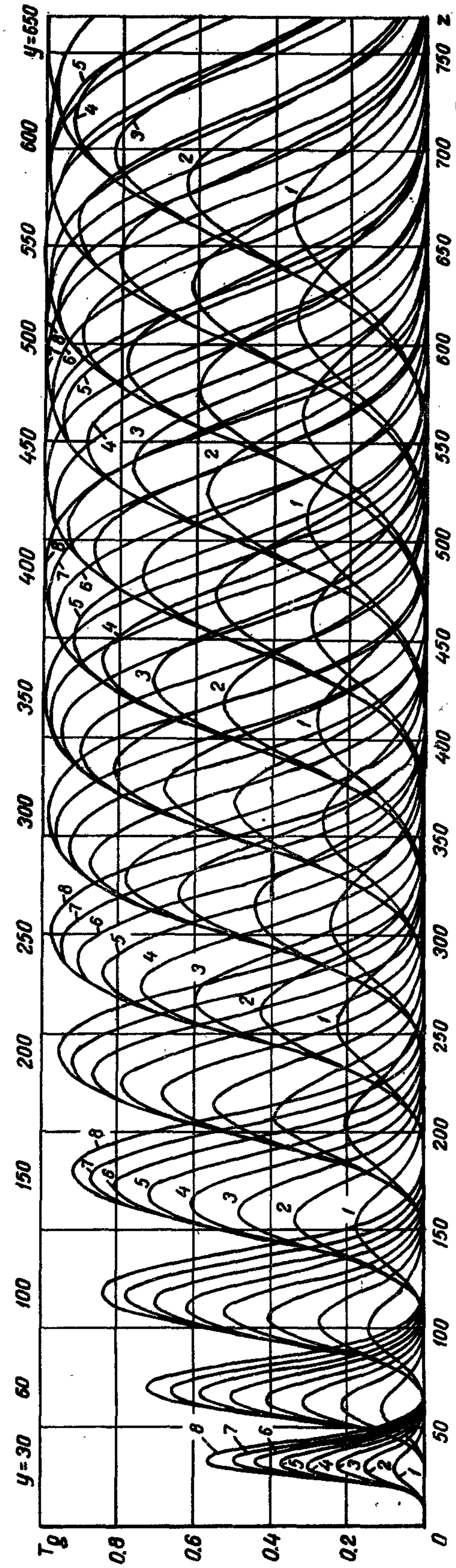
Thus, for the case of heating or cooling of the packing with a constant-temperature gas flow we have

$$T = 0.5(T_g + T_s) = \Phi(\sqrt{2y}) - \Phi(\sqrt{2y} - \sqrt{2z}), \tag{22}$$

and for the case of intermittent pulsed heating or cooling of the packing

$$T = 0.5(T_g + T_s) = \Phi\left(\sqrt{2y} - \sqrt{2(z - z_i)}\right) + \Phi(\sqrt{2z} - \sqrt{2y}). \tag{23}$$

In engineering calculations, constructing the outlet temperature curves and the temperature distribution pattern, it is usual to use not more than twenty points, for example, values of the relative temperature at intervals of 0.05.



Outlet temperature curves for the gas at values of the parameter z_i/y 0.05 (1), 0.10 (2), 0.15 (3), 0.20 (4), 0.25 (5), 0.30 (6), 0.35 (7), 0.40 (8).

At values of $y > 10$ the function $\Phi(2y)^{1/2} \cong 0.5$ and, on the basis of Eq. (22), the outlet temperature curve $T(z, y_0)$ can be calculated for simple heating or cooling of a packing of length y_0 by means of the very simple formula

$$\text{at } T \leqslant 0.5 \quad z = 0.5(\sqrt{2y_0} - a)^2,$$
$$\text{at } T > 0.5 \quad z = 0.5(\sqrt{2y_0} + a)^2, \qquad (24)$$

where the coefficient a is determined from the following data:

T	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
T	0.99	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
a	2.33	1.645	1.28	1.04	0.84	0.675	0.525	0.385	0.253	0.126	0.0

The temperature distribution along the packing at an arbitrary instant of time z_0 is calculated from the equations

$$\text{at } T \leqslant 0.5 \quad y = 0.5(\sqrt{2z_0} + a)^2,$$
$$\text{at } T > 0.5 \quad y = 0.5(\sqrt{2z_0} - a)^2. \qquad (25)$$

For a stepwise change in the gas temperature at the inlet during time z_i the distribution curve for the temperature T over the length of the packing at time z_0 lies between the curves constructed from Eqs. (25) and (26):

$$\text{at } T \leqslant 0.5 \quad y = 0.5(\sqrt{2(z_0 - z_i)} - a)^2,$$
$$\text{at } T > 0.5 \quad y = 0.5(\sqrt{2(z_0 - z_i)} + a)^2. \qquad (26)$$

The maximum temperature at the instant z_0 is reached in the section $y \cong z_0 - 0.5z_i$ and is equal to

$$T_{\max}(z_0, y) \cong \Phi(\sqrt{2z_0} - \sqrt{2z_0 - z_i}) + \Phi(\sqrt{2z_0 - z_i} - \sqrt{2(z - z_i)}). \qquad (27)$$

The maximum value of the temperature T at the outlet from a packing of reduced length y_0 is reached at the instant $z \cong y_0 + 0.5z_i$ and is equal to

$$T_{\max}(z, y_0) \cong \Phi(\sqrt{2y_0} - \sqrt{2y_0 - z_i}) - \Phi(\sqrt{2y_0} - \sqrt{2y_0 + z_i}). \qquad (28)$$

Having the approximate formulas (22) and (23) for determining the arithmetic mean of the temperature of the gas and the packing with an error not greater than 0.0002 and the exact formulas (17) and (18) for determining the difference of these temperatures, we can easily calculate the values of the temperatures T_g and T_s themselves. The function φ can be found from tables [5] or by means of approximate formula (16).

It is clear from (22) that the value of the temperature $T = 0.5$ is reached at a value of y that coincides exactly with z; this is also apparent from the graphs. Consequently, the dimensionless velocity of the heat transfer front in the section with temperature $T = 0.5$, determined as the ratio of the reduced length of the path traversed to the reduced time, is equal to unity. In dimensional quantities we have

$$v_T = \frac{h_g}{h_s(1 - f)} w. \qquad (29)$$

From (25) and (26) it follows that the velocity of propagation of the temperature waves through the packing is directly proportional to the linear gas flow velocity and the ratio of the volume specific heats of gas and packing. As follows from (28), the amplitude of the temperature waves propagating through the packing decreases asymptotically with the length not more rapidly than the difference of the values of the error integral Φ of the arguments $[(2y)^{1/2} - (2y - z_i)^{1/2}]$ and $[(2y)^{1/2} - (2y + z_i)^{1/2}]$.

NOTATION

T is the temperature, x is the coordinate, t is the time, y is the dimensionless length, z is the dimensionless time, k is the heat transfer coefficient, h is the volume specific heat, v is the linear gas velocity, w is the fictitious gas velocity, f is the porosity, $k_1 = k/h_s(1 - f)$; $k_2 = k/h_g f$; $w = vf$; $y = k_2x/v$; $z = k_1(t - x/v)$; $\Phi(x) = [1/(2\pi)^{1/2}] \int_0^x \exp(-t^2/2)\,dt$. Subscripts and superscripts: g stands for gas, s stands for packing, a is the absolute (temperature), 0 is the initial (temperature), i is the time (impulse), during which the gas temperature at the inlet to the bed is different from the initial value.

REFERENCES

1. A. Anzelius, Math. und Mech., 6, no. 4, 291–294, 1926.

2. T. Schumann, J. F. I., 208, no. 1245–29, 403–416.

3. M. Jakob, Heat Transfer [Russian translation], IL, 1960.

4. C. Furnas, Ind. and Eng. Ch., 22, no. 7, 721–731, 1930.

5. L. Karmazina and E. Chistova, Tables of Bessel Functions and Integrals [in Russian], Moscow, 1958.

6 September 1966

Bauman Moscow Higher Technical College